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Physical Consequences of Quantized Fiber-bundled Space

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Considering the motion of charged particles and the charge density in the quantized space, we obtain the effect of charge screening at small distances, which leads to the representation of the vacuum as a medium. A vector potential satisfying the confinement condition of a magnetic monopole is constructed.

1. INTRODUCTION

At present there is no doubt that the interaction mechanism and elementary particle properties are well described in the framework of quantum field theory, which takes into account the quantum properties of such objects. The elementary particles are quantum objects, i.e., they possess a wave character as well as a corpuscular one. Thus, it can be suggested that physical space has quantized (discrete) structure at small distances as well as the elementary particle being in the given space. This space can be not only a four-dimensional, but a many-dimensional or even infinite-dimensional one. The problems concerning the quantized or discrete properties of space have been discussed by Blokhintsev (1973), Prugovečki (1984), Namsrai (1986), and Dineykhan and Namsrai (1985a). In accordance with conceptions developed in the above papers, the geometry of fiber-bundled space is given from the change points of usual space-time on the "internal" space-time (see, for example, Wu and Yang, 1975; Greub and Petry, 1975; Konopleva and Popov, 1980; Daniel and Viallet, 1980) in which operation the gauge group corresponds to arbitrary gauge field. It is known that experimentally observed physical magnitudes are more or less macroscopic and are described well in the usual space, with the exception of some special cases. Therefore our addition characterizing the quantum and fiber-bundle

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properties of the space is a small stochastic perturbation and a magnitude of a higher order than in the usual space. Proceeding from this, Dineykhan and Namsrai (1985a) introduced space quantum properties in the following way, as a small stochastic deviation from the classical coordinates:

$$x^{\mu} \Rightarrow \hat{x}^{\mu} = x^{\mu} + L\Gamma^{\mu}(x) \tag{1}$$

here the x^{μ} are the coordinates of usual (classical) space-time; L is a dimensional parameter characterizing the small scale; $\Gamma^{\mu}(x)$ is determined from the tetrad field $e_a^{\mu}(x)$:

$$\Gamma^{\mu}(x) = \Gamma^{a} e_a^{\mu}(x) \tag{2}$$

and expresses the "internal" space that corresponds to every point of the usual (classical or world) space; and Γ^{a} is the generator of the symmetry group that acts in the internal space. From (1) one can see that the multiplication operation for x^{μ} is noncommutative, i.e.,

$$[\hat{x}^{\nu}, \hat{x}^{\mu}] \neq 0, \qquad \nu \neq \mu \tag{3}$$

Thus (2) and (3) show that the space with the coordinate $\hat{x}^{\mu}$ found in (1) is quantized and fiber-bundled.

We shall use the natural unit system in which the light velocity is equal to unity. Then the invariance of proper time is a fundamental property of quantized as well as classical space,

$$d\tau^2 = g_{\alpha\beta}(x) \cdot dx^{\alpha} \cdot dx^{\beta} = g_{\nu\mu}(\hat{x}) \cdot d\hat{x}^{\mu} \cdot d\hat{x}^{\nu}$$

Here $g_{\alpha\beta}(x)$ and $g_{\nu\mu}(x)$ are the metric tensors in the usual and quantized spaces, respectively. From this property of the space, it follows that the squares of four-dimension velocity are equal in the quantized and usual spaces, i.e.,

$$u_{\nu}(\hat{x}) u^{\nu}(\hat{x}) = u_{\alpha}(x) u^{\alpha}(x) \equiv 1$$

where

$$u_{\nu}(\hat{x}) u^{\nu}(\hat{x}) = g_{\nu\mu}(\hat{x}) u^{\mu}(\hat{x}) u^{\nu}(\hat{x})$$

and

$u^{\mu}(\hat{x}) = d\hat{x}^{\mu}/d\tau$. Then the four-dimension momentum in the space with the quantized coordinates is determined in the following way:

$$p^{\nu}(\hat{x}) = m \cdot u^{\nu}(\hat{x})$$

and its square is the $p_{\nu}(\hat{x}) \cdot p^{\nu}(\hat{x}) = m^2$, where m is the mass of the particle. Dineykhan and Namsrai (1985a) show that the metrical tensor in the quantized and fiber-bundled space consists of two parts, asymmetrical as well as an antisymmetrical one,

$$g^{\nu\mu}(\hat{x}) \equiv g^{\alpha\beta}(x) \frac{\partial \hat{x}^{\nu}}{\partial x^{\alpha}} \frac{\partial \hat{x}^{\mu}}{\partial x^{\beta}}$$

$$= g^{\nu\mu}(x) + L \cdot \Gamma^{a} \left[g^{\alpha\mu}(x) \frac{\partial e_{a}^{\nu}(x)}{\partial x^{\alpha}} + g^{\alpha\nu}(x) \frac{\partial e_{a}^{\mu}(x)}{\partial x^{\alpha}} \right]$$

$$+ 2g^{\alpha\beta}(x) \eta^{a\beta} L^{2} \frac{\partial e_{a}^{\nu}(x)}{\partial x^{\alpha}} \frac{\partial e_{b}^{\mu}(x)}{\partial x^{\beta}}$$

$$+ T_{abc} L^{2} g^{\alpha\beta}(x) \Gamma^{c} \frac{\partial e_{a}^{\nu}(x)}{\partial x^{\alpha}} \frac{\partial e_{b}^{\mu}(x)}{\partial x^{\beta}} \tag{4}$$

and the antisymmetrical part leads to the torsion tensor (Obukhov, 1983; and de Witt, 1984), Here diag $\eta = (+ - - -)$; ν, μ, α, β are the indices of the world (classical) space; a, b, c are the indices of the "internal" space, which is usually antisymmetrical on the permutation of indices. If the addition in (1) is independent of the coordinates of the point, then our space is quantized.

We suppose that the additions to the coordinates in (1), which determine quantized and fiber-bundle properties of the space, are stochastic variables representing the deviation from the observed magnitude. Therefore, the experimentally observed (microscale) values must be average ones; this corresponds to the changeover of the usual space. Then from (1) one can find that the changeover from quantized into the usual (classical) space can be realized in two ways:

1. The classical case, when the dimensional parameter (L) goes to zero
2. We assume that the usual space is some average of the quantized one, i.e.,

$$x^{\mu} = \langle \hat{x}^{\mu} \rangle \equiv \frac{1}{d} Tr(\hat{x}^{\mu})$$

Here the paremeter d is the dimension of the internal space, and the symbol $\langle \cdots \rangle$ means the averaged operation.

In this paper we consider the gauge invariance of the electromagnetic field and the motion of the charged particles in the quantized space. Also it is shown that the unobservability of the magnetic monopoles can be linked with the physics at very small distances, namely with the property of space thus quantized (discretized) as well as fiber-bundled.

2. THE GAUGE INVARIANCE OF THE ELECTROMAGNETIC FIELD

The principles of locality and gauge invariance restrict the form of all interactions independently of their physical nature and thus give the possibility of the construction of a unified consistent theory of interactions among elementary particles. But these principles are well determined in the usual (classical) space. To verify gauge invariance in the case of the electromagnetic field, let us express the four-dimensional potential $A_\mu(\hat{x})$ of the electromagnetic field in the quantized space through the potential $A_\mu(x)$ in the usual space. The interactions are introduced in the following way:

$$\frac{\partial}{\partial \hat{x}^\mu} \Rightarrow \frac{\partial}{\partial \hat{x}^\mu} - qA_\mu(\hat{x})$$

where q is the charge. On the other hand, we know the relation between the divergences in different systems of coordinates,

$$\frac{\partial}{\partial \hat{x}^\mu} = \frac{\partial x^\nu}{\partial \hat{x}^\mu} \frac{\partial}{\partial x^\nu}$$

From these two expressions we obtain

$$\frac{\partial}{\partial \hat{x}^\mu} - qA_\mu(\hat{x}) = \frac{\partial x^\nu}{\partial \hat{x}^\mu}\left[\frac{\partial}{\partial x^\nu} - q \cdot A_\nu(x)\right]$$

and hence

$$A_\mu(\hat{x}) = \frac{\partial x^\nu}{\partial \hat{x}^\mu} A_\nu(x) \tag{5}$$

Thus, we have established the relations between the potentials in the quantized and usual spaces.

In quantum field theory (see, for example, Bogoliubov and Shirkov, 1980) the basic characteristics of the electromagnetic interactions are defined by the field tensor and in our case can be represented in the form

$$F_{\nu\mu}(\hat{x}) = \frac{\partial A_\mu(\hat{x})}{\partial \hat{x}^\nu} - \frac{\partial A_\nu(\hat{x})}{\partial \hat{x}^\mu}$$

Then, taking into account (5) and keeping terms of the order of L^2 only, we have after simple calculations

$$\begin{aligned} F_{\nu\mu}(\hat{x}) = F_{\nu\mu}(x) &+ L\Gamma^a\left[\frac{\partial e_a^\alpha(x)}{\partial x^\nu} F_{\mu\alpha}(x) \right. \\ &+ \left.\frac{\partial e_a^\alpha(x)}{\partial x^\mu} F_{\alpha\nu}(x)\right] + L^2\Gamma^a\Gamma^b\left[\frac{\partial e_a^\alpha(x)}{\partial x^\lambda}\frac{\partial e_b^\lambda(x)}{\partial x^\nu} F_{\alpha\mu}(x)\right. \\ &+ \left.\frac{\partial e_a^\alpha(x)}{\partial x^\lambda}\frac{\partial e_b^\lambda(x)}{\partial x^\mu} F_{\nu\alpha}(x) + \frac{\partial e_a^\alpha(x)}{\partial x^\nu}\frac{\partial e_b^\beta(x)}{\partial x^\mu} F_{\alpha\beta}(x)\right] \end{aligned} \tag{6}$$

Here $F_{\alpha\beta}(x)$ is the electromagnetic field tensor in the usual space,

$$F_{\alpha\beta}(x)=\frac{\partial A_\beta(x)}{\partial x^\alpha}-\frac{\partial A_\alpha(x)}{\partial x^\beta}$$

and is invariant under the gauge transformations of the electromagnetic field $A_\mu(x)\Rightarrow A_\mu(x)+\partial\mu f(x)$, where $f(x)$ is an arbitrary scalar function in the usual space. Then it can be seen from (6) that the field tensor $F_{\nu\mu}(\hat{x})$ defined in the quantized space is also invariant under this gauge transformation. The Lagrangian of the electromagnetic field is written in the standard form:

$$\mathscr{L}(\hat{x})=-\tfrac{1}{4}F_{\nu\mu}(\hat{x})F^{\nu\mu}(\hat{x})$$

and is undoubtedly invariant under the gauge transformations of the electromagnetica field $A_\mu(x)$.

Now let us study the expressions for the electromagnetic field of the charge q in the space with quantized coordinates. From (5) and taking into account only the order L, we have, after simple calculations,

$$qA_\mu(\hat{x})=qA_\mu(x)+qL\Gamma^a\frac{\partial e_a^\nu(x)}{\partial x^\mu}A_\nu(x) \tag{7}$$

If the "internal" space is a colored one, then the second term in (7) can be considered as the color component $B_\mu^a(x)$ of the field, which is defined in the form

$$g_c B_\mu^a(x)=qL\frac{\partial e_a^\nu(x)}{\partial x^\mu}A_\nu(x)$$

where g_c is the coupling constant and Γ^a is the generator.

Thus, according to this scheme, the electromagnetic field consists of two parts: the color octet as well as color singlet. Though the color components of the electromagnetic field do not give any contribution to the one-photon exchange interactions, they contribute in a not dominating way to two-photon interactions. Ignatiev et al. (1981) suggested that generally the electromagnetic field consists of two parts: the color octet as well as color singlet. Proceeding from this hypothesis, the integer charge of quarks and the dynamics of spontaneously broken color symmetry can be explained. From the point of view of our scheme the nature of the color component of the electromagnetic field or the dynamics of spontaneously broken color symmetry, i.e., the properties of the "colored" photon, can be linked to the quantized as well as fiber-bundle characteristics of space at very small distances.

3. THE DENSITY OF CHARGE IN THE QUANTIZED SPACE

Consider the vector current of the electromagnetic field. The four-dimensional current of the electromagnetic field is defined from the field tensor in standard form (see, for example, Felsager, 1981):

$$j_\nu(\hat{x}) = \frac{\partial F_{\nu\mu}(\hat{x})}{\partial \hat{x}^\mu}$$

Making the necessary calculation and taking into account (6), we have for the current of the electromagnetic field in the quantized space

$$\begin{aligned} j_\nu(\hat{x}) = j_\nu(x) + L\Gamma^a \bigg[& \frac{\partial e_a^\lambda(x)}{\partial x^\mu} \frac{\partial F_{\lambda\nu}(x)}{\partial x^\mu} \\ & + \frac{\partial e_a^\lambda(x)}{\partial x^\nu} \frac{\partial F_{\mu\lambda}(x)}{\partial x^\mu} + \frac{\partial e_a^\lambda(x)}{\partial x^\mu} \frac{\partial F_{\mu\nu}(x)}{\partial x^\lambda} \bigg] \\ & + L^2\Gamma^a\Gamma^b \bigg(\frac{\partial e_a^\lambda(x)}{\partial x^\mu} \frac{\partial e_b^s(x)}{\partial x^\nu} \frac{\partial F_{s\mu}(x)}{\partial x^\lambda} + \frac{\partial e_a^\lambda(x)}{\partial x^\mu} \\ & \times \frac{\partial e_b^s(x)}{\partial x^\mu} \frac{\partial F_{\nu s}(x)}{\partial x^\lambda} + \frac{\partial e_a^s(x)}{\partial x^\lambda} \frac{\partial e_b^\lambda(x)}{\partial x^\nu} \frac{\partial F_{s\mu}(x)}{\partial x^\mu} \\ & + \frac{\partial e_a^s(x)}{\partial x^\lambda} \frac{\partial e_b^\lambda(x)}{\partial x^\mu} \frac{\partial F_{\nu s}(x)}{\partial x^\mu} + \frac{\partial e_a^\lambda(x)}{\partial x^\nu} \frac{\partial e_b^s(x)}{\partial x^\mu} \\ & + \frac{\partial F_{\lambda s}(x)}{\partial x^\mu} + \frac{\partial e_a^s(x)}{\partial x^\lambda} \frac{\partial e_b^\lambda(x)}{\partial x^\mu} \frac{\partial F_{\nu\mu}(x)}{\partial x^s} \bigg) \end{aligned} \tag{8}$$

where $j_\nu(x)$ is the current and $f_{\nu\mu}(x)$ is the field tensor in the usual space, Γ^a is the generator of the symmetry group in the "internal" space, and $e_a^\nu(x)$ is the tetrad field.

Now let us study the charge density in the quantized space. Usually, the charge density is determined as the zero component of the four-dimensional current $j_0(x)$ of the electromagnetic field. Averaging (8), we have

$$\begin{aligned} \langle \rho(\hat{x}) \rangle & \equiv \langle j_0(\hat{x}) \rangle \\ & = \rho(x) + L^2 \bigg[\frac{\partial e_a^i(x)}{\partial x^m} \frac{\partial e_a^j(x)}{\partial x^m} + \frac{\partial e_a^j(x)}{\partial x^m} \frac{\partial e_a^m}{\partial x^i} \\ & \quad + \frac{\partial e_a^c(x)}{\partial x^m} \frac{\partial e_a^m(x)}{\partial x 1^j} \bigg] \frac{\partial F_{0j}(x)}{\partial x^i} + L^2 \bigg[\frac{\partial e_a^i(x)}{\partial x^m} \frac{\partial e_a^j(x)}{\partial x^n} \\ & \quad + \frac{\partial e_a^j(x)}{\partial x^n} \frac{\partial e_a^m(x)}{\partial x^i} \bigg] v^n(x) \frac{\partial F_{jm}(x)}{\partial x^i} + L^2 \frac{\partial e_a^j(x)}{\partial x^i} \\ & \quad \times \frac{\partial e_a^i(x)}{\partial x^n} v^n(x) \frac{\partial F_{jm}(x)}{\partial x^m} \end{aligned} \tag{9}$$

where i, j, n, m, and $a=1,2,3$; $F_{mn}(x)$ is the field tensor and $v^n(x)$ is the velocity of the particles in the usual space. From (9) one can see that the charge density has additional terms characterizing the quantized and fiber-bundled properties of space at small distances. To understand the physical nature of this additional term in (9), knowledge of the concrete dependence of the tetrad field $e_a^\mu(x)$ on coordinates x, i.e., the structure of "internal" space, is necessary. Dineykhan and Nansrai (1985a) suppose that the structure might be linked with the physical nature of the magnetic Dirac monopoles, and in the next section we will discuss this in detail. Consider the "internal" space as a string (maybe the Dirac string) directed along the axis Z; then $e_a^m(x)$ takes the form

$$e_a^m(x)=\begin{pmatrix} x/r & -y/r & 0 \\ y/r & x/r & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad r=(x^2+y^2)^{1/2} \tag{10}$$

Further expressing the field tensor $F_{\nu\mu}(x)$ through the electrical $\mathbf{E}$ and the magnetic $\mathbf{H}$ strengths of the electromagnetic field and after the necessary calculation, we obtain

$$\begin{aligned}\rho_{\text{qua}}(x)=\langle\rho(\hat{x})\rangle &= -(1+2L^2/r^2)\operatorname{div}\mathbf{E} \\ &\quad -\operatorname{div}[(L^2/r^2)\mathbf{E}-2\mathbf{n}(\mathbf{n}\cdot\mathbf{E})L^2/r^2]\end{aligned} \tag{11}$$

where ρ_{qua} and $\rho(x)=-\operatorname{div}\mathbf{E}$ are the charge density in quantized and usual spaces, respectively, and $\mathbf{n}$ is the unit vector directed along the string radius r. Let us consider each term of (11) separately.

Consider as a source of electromagnetic field a particle with charge q and mass m. Then the variabie

$$q(r)=q(1+2L^2/r^2) \tag{12}$$

is the charge with the density $\rho(x)$ in the quantized space. It can be seen from (12) that the value of the charge $q(r)$ depends on the distance; for $r<L$ it increases with the decrease of r, thus leading to the effect of screening the charge. On the other hand, from grand unification theory (see, for example, Ellis, 1984) it is known that the coupling constant or the electric charge increases at small distances and changes to the regime of strong coupling. It can be confirmed that the dynamics of increase of the coupling constant of the electromagnetic interaction can be linked with the properties of space as quantized and fiber-bundled. So it was considered that the gauge field in the quantized space corresponds to the symmetry group $U(1)$, i.e., there are the usual electromagnetic interactions of the electron, and we have

obtained the effect of charge screening at small distances. If we consider the gauge field in the quantized space defined by non-Abelian symmetry group, i.e., the interaction amont colored quarks, this can lead to the effect of antiscreening of the colored charge at small distances, i.e., to the asymptotic freedom. This problem deserves further investigation.

Let us consider the second term in (11). This is the divergence of the vector **P**, which is defined in the following way:

$$\mathbf{P}=\mathbf{E}-2\mathbf{n}(\mathbf{n}\cdot\mathbf{E})]L^2/r^2$$

where **E** is the electric field strength, L is the dimensional parameter, and **n** is the unit vector directed along the string radius. Introduce the symmetrical tensor α_{ij} of the determination in the following form:

$$\alpha_{ij}=L^2(2n_in_j-\delta_{ij})[\mathbf{n}\cdot\mathrm{grad}(1/r)] \tag{13}$$

Using (13), we obtain for the vector **P**

$$P_i=\alpha_{ij}E_j \tag{14}$$

From the electrodynamics of continuous media (see, for example, Landau and Lifschitz, 1982) it is known that the vector **P** defined in the form (14) through the field strength **E** is the electric dipole moment of the medium and the symmetrical tensor α_{ij} defined in (13) is the polarization vector of the medium. Then the electric dipole moment **P** leads to the additional density of charge $\rho'(x)$, which is determined in the form $\rho'(x)=-\mathrm{div}\,\mathbf{P}$. From this one can see that the quantized and fiber-bundled space is like a dielectric medium, i.e., the model suggests that to every point of the world space there corresponds a string, naturally leading to a representation of the vacuum as the medium.

4. THE MOTION OF CHARGED PARTICLES IN THE ELECTROMAGNETIC FIELD

Now we consider the motion of charged particles in the quantized space. A particle carrying the charge q in the electromagnetic field will experience a force **F**, written in the standard form, which depends on the field tensor defined in (6): $F_\nu(\hat{x})=qF_{\nu\mu}(\hat{x})u^\mu(\hat{x})$, where $u^\mu(\hat{x})=u^\mu(x)+Lu^\nu(x)\,\partial\Gamma^\mu(x)/\partial x^\nu$ and $u^\nu(x)$ are the four-dimensional velocity of particles in the quantized and usual spaces, respectively; $\Gamma^\mu(x)$ is defined in (2). After averageing and some calculations we have

$$\langle F_\nu(\hat{x})\rangle=qF_{\nu\mu}(x)u^\mu(x)$$
$$+L^2q\frac{\partial e_a^\lambda(x)}{\partial x^s}\frac{\partial e_a^s(x)}{\partial x^\nu}F_{\lambda\mu}(x)u^\mu(x)$$

where $F_{\nu\mu}(x)$ is the field tensor in the usual space, and $e_a^\mu(x)$ is the tetrad field. For further calculations knowledge of the structure of the "internal" space, i.e., the concrete dependence of the tetrad field $e_a^\mu(x)$ on coordinates x, is necessary. If the tetrad field $e_a^\mu(x)$ is defined as in (10), then after simple calculation we obtain finally

$$\begin{aligned}\mathbf{F} &= q(1+L^2/2r^2)\mathbf{E} + q(1+L^2/r^2)[\mathbf{v}\times\mathbf{H}] \\ &\quad + q(L^2/2r^2)[\mathbf{E} - 2\mathbf{n}(\mathbf{n}\cdot\mathbf{E})] + q(L^2/r^2)\mathbf{n}([\mathbf{n}\times\mathbf{H}]\cdot\mathbf{v}) \qquad (15)\end{aligned}$$

where q is the charge of the particle, $\mathbf{E}$ is the electrical and $\mathbf{H}$ the magnetic strengths of the electromagnetic field, respectively, L is the dimensional parameter, and $\mathbf{n}$ is the unit vector directed along the string radius. The first term of (15) is the usual Coulomb force with the charge $q(1+L^2/2r^2)$, and the second term is the Lorentz force with the charge $q(1+L^2/r^2)$. The third term represents the contribution of the electric dipole moment determined by (14) and the fourth one the contribution of the magnetic dipole moment (see, for example, Landau and Lifschitz, 1982), which is linked to the vortex motion. Then from (15) one can see that the motion of charged particles in the quantized space accompanied by the electromagnetic field is equivalent to the motion of particles in a dielectric medium.

5. THE CONFINEMENT OF MAGNETIC MONOPOLE AND THE STRUCTURE OF "INTERNAL" SPACE

In the preceding section we have shown that the quantized and fiber-bundled properties of the space lead to representation of the vacuum as a medium, i.e., in a vacuum a particle–antiparticle pair can be born, which leads to the effect of screening the charge. Dineykhan and Namsrai (1985a) attained the nonsingular potential (of course, except for the singularity at the origin of the coordinate system) for the magnetic monopole, but this does not explain the unobservability of this object. On the other hand, although the magnetic monopole is not experimentally observed, interest in this object has never been so great as at the present time. This has its origin in the papers by t'Hooft (1979) and Polyakov (1979), in which they showed that the magnetic monopole inevitably arises in some theories of the gauge field. We try to demonstrate that the absence of the magnetic monopole can be connected to the physics at very small distances, namely to such properties of space as the quantized as well as fiber-bundled ones. Dineykhan and Namsrai (1985a) demonstrated the possibility of choosing the vector potential such that its rotor would be equal to the field of the magnetic monopole at the simultaneous condition that the field possesses a nonzero divergence, i.e., the density of the magnetic monopole is equal to

$$\rho_m(x) \equiv \langle \operatorname{div} \mathbf{H}(\hat{x}) \rangle = L^2 \varepsilon^{ijk} \left[\frac{\partial \Gamma^n(x)}{\partial x^i} \frac{\partial \Gamma^m(x)}{\partial x_j} \frac{\partial^2 A_k(x)}{\partial x^n \, \partial x^m} \right]$$

where $A_k(x)$ is the vector potential of the monopole, $\Gamma^m(x) = \Gamma^a e_a^m(x)$ is defined as in (2), and L is the dimensional parameter. Now we choose the vector potential of the monopole in the form

$$\mathbf{A}(x) = \frac{3g}{8} \frac{\mathbf{\Gamma}}{L} \ln\left(\frac{r}{L}\right), \qquad r = (x^2 + y^2 + z^2)^{1/2}$$

where g is the magnetic charge, and $\mathbf{\Gamma}$ is the generator of the symmetry group effective in the "internal" space; for concrete calculations we should be using the generator of the group $SU(2)$, i.e., $\mathbf{\Gamma} = i\boldsymbol{\sigma}$, where $\boldsymbol{\sigma}$ is the Pauli matrix. After the necessary calculations, we obtain for the density of magnetic charge

$$\rho_m(x) = \frac{3Lg}{r^4} \left(\frac{\partial e_a^n(x)}{\partial x^a} \frac{\partial e_b^m(x)}{\partial x^b} - \frac{\partial e_a^n(x)}{\partial x^b} \frac{\partial e_b^m(x)}{\partial x^a} \right) x^n x^m \tag{17}$$

From (17) one can see that for further calculations knowledge of the structure of "internal" space, i.e., the concrete dependence of the tetrad field $e_a^m(x)$ on coordinates x, is necessary. Choose for the "internal" space a spherical coordinate system; then for $e_a^m(x)$ we get

$$e_a^m(x) = \begin{pmatrix} x/r & zx/r\rho & -y/r \\ y/r & zy/r\rho & x/r \\ z/r & -\rho/r & 0 \end{pmatrix} \tag{18}$$

where $\rho = (x^2 + y^2)^{1/2}$, $r = (x^2 + y^2 + z^2)^{1/2}$, and the density $\rho_m(x)$ of magnetic charge g for the above choice of the "internal" space and defined in (17) is not equal to zero and its volume integral is equal to $4\pi g$. Nevertheless, magnetic charge with the density $\rho_m(x)$ as defined in the (17) is experimentally unobservable because the potential determined in (16) at large distances increases logarithmically, satisfying the confinement condition of the magnetic monopole.

With the help of the tetrad field $e_a^\mu(x)$ one can define the metric tensor $g^{\nu\mu}(x)$ in the following way (see, for example, De Witt, 1984):

$$g^{\nu\mu}(x) = e_a^\nu(x) e_b^\mu(x) \eta^{\alpha\beta}$$

where η^{ab} is the metric tensor in the usual Euclidean space, i.e., diag $\eta = (+----)$. With the tetrad field $e_a^\mu(x)$ defined in (16) and (18), the metric tensor $g^{\nu\mu}(x)$ determined in (19) is equal to the tensor η^{ab}. Then the transformation of covariant and contravariant components of the vector from one to another with the help of the metric tensor $g^{\nu\mu}(x)$ is not essential for concrete calculations. So in concrete calculations we did not take the metric tensor explicitly. The potential defined in (16) is nonsingular and has the same form in all regions of space, which is one of the advantages over other nonsingular potentials (see, for example, Wu and Yang, 1976).

Now let us study the case when the "internal" space represents strings directed along the axis Z, i.e., the tetrad field $e_a^\mu(x)$ defined in (10). Then the density $\rho_m(x)$ of the magnetic monopole determined in (17) for arbitrary nonsingular potentials is identically equal to zero, i.e., the flux of magnetic charge is absent. In this case for the existence of the monopole or the magnetic charge flux a singular potential is necessary as well as the Dirac potential, which is defined in the pioneering paper by Dirac (1931). By such methods the unobservability of the magnetic monopole, as shown by Dirac, has its origin in the condition of quantization of the magnetic charge itself.

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